Number Theory Seminar

NMAG470

November 5 at 15:40 in K9

The finiteness theorem for universal m-gonal forms

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A (generalized) m-gonal number is a non-negative rational integer defined by $P_m(x) := \frac{(m-2)x^2-(m-4)x}{2}$ where $x \in \mathbb{Z}$. We call $F_m(\mathbf{x}) = \sum_{i=1}^{n} a_i P_m(x_i)$ where $a_i \in \mathbb{N}$ as m-gonal form. Kane proved that there always exists a (unique, minimal) $\gamma_m \in \mathbb{N}$ such that if an m-gonal form $F_m(\mathbf{x})$ represents every positive rational integer up to γ_m, then $F_m(\mathbf{x})$ is universal, i.e., $F_m(\mathbf{x})$ represents every positive rational integer. There are some examples for γ_m which are concretely calculated for some small m's: $\gamma_3 = \gamma_6 = 8$ [Bosma and Kane], $\gamma_4 = 15$ [15-Theorem, Conway and Schneeberger], $\gamma_5 = 109$ [Ju] and $\gamma_8 = 60$ [Ju and Oh]. The growth of γ_m(since $m - 4 \leq \gamma_m$ for $6 \leq m$, the γ_m asymptotically increases as m increases) was firstly questioned by Kane and Liu who proved that for $m \geq 3$ and every $\epsilon > 0$, there exists an absolute (effective) constant C_ϵ such that $m - 4 \leq \gamma_m \ll C_\epsilon m^{7+\epsilon}$.

In this talk, we will show that for $m \geq 3$, there is an absolute constant C such that $m - 4 \leq \gamma_m \leq C \cdot m$, which implies that the growth of γ_m is exactly linear on m.

This is a joint work with B. M. Kim.

Web semináře:
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