

Bootstrapping of M-smoothers

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ROBUST 2010

Hora Matky Boží, Králiky, 1st - 5th of February



JELEC TLOUŠŤ

(Leuciscus cephalus; L., 1758)

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- Name: 3373



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- Name: 3373 → no nick name... **BUT...**



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- Weight: 1.390 grams



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- Length: 48 millimeters (*length range within 47 - 66*)



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Observations taken in a two months trial from February 3rd till March 12th in 2009

- Name: 3373 → no nick name... BUT...
- Weight: 1.390 grams (*weight range within 1.390 - 4.735*)
- Length: 48 millimeters (*length range within 47 - 66*)



Fish specific activity vs. its weight (length)

Design of Experiment:

- **independent** observations with **heteroscedastic** variance structure;



Fish specific activity vs. its weight (length)

Design of Experiment:

- independent observations with heteroscedastic variance structure;
- **technical background of the experiment**



Fish specific activity vs. its weight (length)

Design of Experiment:

- independent observations with heteroscedastic variance structure;
- technical background of the experiment \Rightarrow possible **outlier observations**;



Fish specific activity vs. its weight (length)

Design of Experiment:

- independent observations with heteroscedastic variance structure;
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- evolution progress



Fish specific activity vs. its weight (length)

Design of Experiment:

- independent observations with heteroscedastic variance structure;
- technical background of the experiment \Rightarrow possible outlier observations;
- evolution progress \Rightarrow possibility of a **sudden change** in behaviour;



Fish specific activity vs. its weight (length)

Design of Experiment:

- independent observations with heteroscedastic variance structure;
- technical background of the experiment \Rightarrow possible outlier observations;
- evolution progress \Rightarrow possibility of a sudden change in behaviour;

Statistical model behind:

- **Local polynomial modelling** with **robust approach**



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Statistical model behind:

- Local polynomial modelling with robust approach \Rightarrow **M-smoothers**;



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- Local polynomial modelling with robust approach \Rightarrow M-smoothers;
- Testing for a **change-point occurrence**



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Design of Experiment:

- independent observations with heteroscedastic variance structure;
- technical background of the experiment \Rightarrow possible outlier observations;
- evolution progress \Rightarrow possibility of a sudden change in behaviour;

Statistical model behind:

- Local polynomial modelling with robust approach \Rightarrow M-smoothers;
- Testing for a change-point occurrence \leftarrow given a **specific weight-point**;



Asymptotics & bootstrapping

- asymptotic normality of the test statistic under the null hypothesis;



Asymptotics & bootstrapping

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$$\sqrt{Nh_N} \cdot \left| \widehat{m}_+^{(\nu)}(x_0) - \widehat{m}_-^{(\nu)}(x_0) \right| \xrightarrow{D} N \left(0, \frac{c_{x_0}^2 \mathbb{E} \psi^2(\epsilon \cdot \sigma(x_0))}{f(x_0)} \cdot \vec{e}_{\nu+1}^\top h_N^\top S_1^{-1} S_2 S_1^{-1} h_N \vec{e}_{\nu+1} \right)$$



Asymptotics & bootstrapping

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- the asymptotic distribution heavily depends on some unknown quantities...



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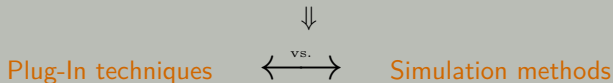


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Plug-In techniques



Simulation methods

- bootstrapping of M-smoothers based on a heteroscedastic model...



Bibliography (a short overview)



A. S. Harestad & F. L. Bunnell (1979).
Home range and body weight - a re-evaluation.
Ecology, 60: 389–402.



S. Nakano (1995).
Individual differences in resource use, growth and emigration
under influence of a dominance hierarchy in fluvial red-spotted
masu salmon in a natural habitat.
Journal of Animal Ecology, 64: 75–84.



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Thank you!

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