Generalized Lehmann's Alternative model and skew symmetry: Supplement to the lecture

G(x:δ,F)=h(F(x): δ)

g(x: δ,F)=f(x)2F(δx),

Or =f(x)2K(δx)

Thus

$$h(t:\delta,F,K)=\int_0^t 2K(\delta F^{-1}(u))du$$

and

$$h(F(x):\delta,F,K)=\int_0^{F(x)} 2K(\delta F^{-1}(u))du$$

In my lecture, the setting

$h(F(x):\delta)=F(x)2F(\delta x)$ or $h(F(x):\delta)=F(x)2K(\delta x)$,

where F and K are cumulative distribution functions,

was not proper since it approaches to 2 as $x \to \infty$.

I took a look into a paper by Azzalini (2005, Scandinavian J. of Statistics)

to see their definition of skew symmetry.

There the definition was given in a form of a density function,

$f(x)2F(\delta x)$,or $f(x)2K(\delta x)$,

not in a form of a cumulative distribution function.

Now in order to see the correspondence of a skew symmetry to

our GLA model(or a transformation model), we can set

$G(x:\delta,F)=h(F(x):\delta)$ in the following way,

where F is an original distribution and G is a transformed distribution

The density function is,

$$g(x:\delta,F)=\frac{d}{dx}h(F(x):\delta) = h'(F(x):\delta)f(x), \text{ (where } h'(t:\delta)=\frac{d}{dt}h(t:\delta))$$

where we see $h'(F(x):\delta)$ is acting on f(x) to give it a "skew"

as $2F(\delta x)$ or $2K(\delta x)$ does in their skew symmetric models.

Therefore, the proper setting for our GLA model to respond

to their skew symmetry, will be

$$h(t:\delta,F)=\int_0^t 2F(\delta F^{-1}(u))du \text{ or } =\int_0^t 2K(\delta F^{-1}(u))du, \text{ and}$$

$$G(x:\delta,F) \equiv h(F(x):\delta)=\int_0^{F(x)} 2F(\delta F^{-1}(u))du \text{ or } =\int_0^{F(x)} 2K(\delta F^{-1}(u))du.$$

Now, what we need to check is that thus specified $h(t:\delta,F)$ satisfies

the general condition as was done in our Homework 4.

These conditions for $h(t:\delta)$ can be checked for the regions:

$\{\delta > 0, 0 < t < \frac{1}{2}\}, \{\delta > 0, \frac{1}{2} < t < 1\}$ and for δ<0 regions respectively.